Задача 11. Дана функція $u(M)=u(x,y,z)$ і точки M_1 і M_2. Обчислити похідну цієї функції в точці M_1 за напрямом вектора $\overline{M_1M_2}$ та $\overrightarrow{grad}\ u(M_1)$.

1. $u(M)=x^2y+y^2z+z^2x,$ $M_1(1,-1,2),\ M_2(3,4,-1)$
2. $u(M)=5xy^3z^2,$ $M_1(2,1,-1),\ M_2(4,-3,0)$
3. $u(M)=ln(x^2+y^2+z^2),$ $M_1(-1,2,1),\ M_2(3,1,-1)$
4. $u(M)=z\cdot exp\left(x^2+y^2+z^2\right),$ $M_1(0,0,0),\ M_2(3,-4,2)$
5. $u(M)=ln(xy+yz+xz),$ $M_1(-2,3,-1),\ M_2(2,1,-3)$
6. $u(M)=\sqrt{1+x^2+y^2+z^2},$ $M_1(1,1,1),\ M_2(3,2,1)$
7. $u(M)=x^2y+xz^2-2,$ $M_1(1,1,-1),\ M_2(2,-1,3)$
8. $u(M)=x\cdot e^y+ye^x-z^2,$ $M_1(3,0,2),\ M_2(4,1,3)$
9. $u(M)=3xy^2+z^2-xyz,$ $M_1(1,1,2),\ M_2(3,-1,4)$
10. $u(M)=5x^2yz-xy^2z+yz^2,$ $M_1(1,1,1),\ M_2(9,-3,9)$
11. $u(M)=x:(x^2+y^2+z^2),$ $M_1(1,2,2),\ M_2(-3,2,-1)$
12. $u(M)=y^2z-2xyz+z^2,$ $M_1(3,1,-1),\ M_2(-2,1,4)$
13. $u(M)=x^2+y^2+z^2-2xyz,$ $M_1(1,-1,2),\ M_2(5,-1,4)$
14. $u(M)=ln(1+xy^2+z^2),$ $M_1(1,1,1),\ M_2(3,-5,1)$
15. $u(M)=x^2+2y^2-4z^2-5,$ $M_1(1,2,1),\ M_2(-3,-2,6)$
16. $u(M)=ln(x^3+y^3+z+1),$ $M_1(1,3,0),\ M_2(-4,1,3)$
17. $u(M)=x-2y+e^z,$ $M_1(-4,-5,0),\ M_2(2,3,4)$
18. $u(M)=x^y-3xyz,$ $M_1(1,-1,2),\ M_2(3,4,-1)$
19. $u(M)=3x^2yz^3,$ $M_1(1,-1,2),\ M_2(3,4,-1)$
20. $u(M)=exp\left(xy+z^2\right),$ $M_1(-5,0,2),\ M_2(2,4,-3)$
21. $u(M)=x^{yz},$ $M_1(3,1,4),\ M_2(1,-1,-1)$
22. $u(M)=\left(x^2+y^2+z^2\right),$ $M_1(1,2,-1),\ M_2(0,-1,3)$
23. $u(M)=\left(x-y\right)^z,$ $M_1(1,5,0),\ M_2(3,7,-2)$
24. $u(M)=x^2y+y^2z-3z,$ $M_1(0,-2,-1),\ M_2(12,-5,0)$
25. $u(M)=10:(x^2+y^2+z^2+1),$ $M_1(-1,2,-2),\ M_2(2,0,1)$
26. $u(M)=ln(1+x^2-y^2+z^2),$ $M_1(1,1,1),\ M_2(5,-4,8)$
27. $u(M)=xy^{-1}+yz^{-1}-zx^{-1},$ $M_1(-1,1,1),\ M_2(2,3,4)$
28. $u(M)=x^3+xy^2-6xyz,$ $M_1(1,3,-5),\ M_2(4,2,-2)$
29. $u(M)=xy^{-1}-yz^{-1}-xz^{-1},$ $M_1(2,2,2),\ M_2(-3,4,1)$
30. $u(M)=exp\left(x-yz\right),$ $M_1(1,0,3),\ M_2(2,-4,5)$

Задача 12. Обчислити поверхневий інтеграл першого роду по поверхні *S*, якщо *S* – частина площини *(p)*, яка відсічена координатними площинами.

1. $\iint\limits_s(2x+3y+2z)ds,$ $(p):\ x+3y+z=3$
2. $\iint\limits_s(2+y-7x+9z)ds,$ $(p):\ 2x-y-2z=-2$
3. $\iint\limits_s(6x+y+4z)ds,$ $(p):\ 3x+3y+z=3$
4. $\iint\limits_s(x+2y+3z)ds,$ $(p):\ x+y+z=2$

5. $\iint_s (3x-2y+6z)ds,$ $(p):\ 2x+y+2z=2$

6. $\iint_s (5x-8y-z)ds,$ $(p):\ 2x-3y+z=6$

7. $\iint_s (2+y-7x+9z)ds,$ $(p):\ x+3y+z=3$

8. $\iint_s (3y-x-z)ds,$ $(p):\ x-y+z=2$

9. $\iint_s (3y-2x-2z)ds,$ $(p):\ 2x-y-2z=-2$

10. $\iint_s (2x-3y+z)ds,$ $(p):\ x+2y+z=2$

11. $\iint_s (5x+y-z)ds,$ $(p):\ x+2y+2z=2$

12. $\iint_s (3x+2y+2z)ds,$ $(p):\ 3x+2y+2z=6$

13. $\iint_s (2x+3y-z)ds,$ $(p):\ 2x+y+z=2$

14. $\iint_s (9x+2y+z)ds,$ $(p):\ 2x+y+z=4$

15. $\iint_s (3x+8y+8z)ds,$ $(p):\ x+4y+2z=8$

16. $\iint_s (4y-x+4z)ds,$ $(p):\ x-2y+2z=2$

17. $\iint_s (7x+y+2z)ds,$ $(p):\ 3x-2y+2z=6$

18. $\iint_s (2x+3y+z)ds,$ $(p):\ 2x+3y+z=6$

19. $\iint_s (4x-y+z)ds,$ $(p):\ x-y+z=2$

20. $\iint_s (6x-y+8z)ds,$ $(p):\ x+y+2z=2$

21. $\iint_s (4x-4y-z)ds,$ $(p):\ x+2y+2z=4$

22. $\iint_s (2x+5y+z)ds,$ $(p):\ x+y+2z=2$

23. $\iint_s (4x-y+4z)ds,$ $(p):\ 2x+2y+z=4$

24. $\iint_s (5x+2y+z)ds,$ $(p):\ x+2y+z=2$

25. $\iint_s (2x+5y+10z)ds,$ $(p):\ 2x+y+3z=6$

26. $\iint_s (2x+15y+z)ds,$ $(p):\ x+2y+2z=2$

27. $\iint_s (3x+10y-z)ds,$ $(p):\ x+3y+2z=6$

28. $\iint_s (2x+3y+z)ds,$ $(p):\ 2x+2y+z=2$

29. $\iint_s (5x-y+5z)ds,$ $(p):\ 3x+2y+z=6$

30. $\iint_s (x+3y+2z)ds,$ $(p):\ 2x+y+2z=2$

Задача 13. Обчислити двома способами потік векторного поля $\overline{a}(M)$ через зовнішню поверхню піраміди, утвореною площиною (p) і координатними площинами: а) використовуючи його означення; б) за допомогою формули Остроградського – Гаусса.

1. $\overline{a}(M)=3x\overline{i}+(y+z)\overline{j}+(x-z)\overline{k},\qquad (p):\ x+3y+z=3$
2. $\overline{a}(M)=(3x-1)\overline{i}+(y-x+z)\overline{j}+4z\overline{k},\qquad (p):\ 2x-y-2z=2$
3. $\overline{a}(M)=x\overline{i}+(x+z)\overline{j}+(y+z)\overline{k},\qquad (p):\ 3x+3y+z=3$
4. $\overline{a}(M)=(x+z)\overline{i}+(z-x)\overline{j}+(x+2y+z)\overline{k},\qquad (p):\ x+y+z=2$
5. $\overline{a}(M)=(y+2z)\overline{i}+(x+2z)\overline{j}+(x-2y)\overline{k},\qquad (p):\ 2x+y+2z=2$
6. $\overline{a}(M)=(x+z)\overline{i}+2y\overline{j}+(x+y-z)\overline{k},\qquad (p):\ x+2y+z=2$
7. $\overline{a}(M)=(3x-y)\overline{i}+(2y+z)\overline{j}+(2z-x)\overline{k},\qquad (p):\ 2x-3y+z=6$
8. $\overline{a}(M)=(2y+z)\overline{i}+(x-y)\overline{j}-2z\overline{k},\qquad (p):\ x-y+z=2$
9. $\overline{a}(M)=(x+y)\overline{i}+3y\overline{j}+(y-z)\overline{k},\qquad (p):\ 2x-y-2z=-2$
10. $\overline{a}(M)=(x+y-z)\overline{i}-2y\overline{j}+(x+2z)\overline{k},\qquad (p):\ x+2y+z=2$
11. $\overline{a}(M)=(y-z)\overline{i}+(2x+y)\overline{j}+z\overline{k},\qquad (p):\ 2x+y+z=2$
12. $\overline{a}(M)=x\overline{i}+(y-2z)\overline{j}+(2x-y+2z)\overline{k},\qquad (p):\ x+2y+2z=2$
13. $\overline{a}(M)=(x+2z)\overline{i}+(y-3z)\overline{j}+z\overline{k},\qquad (p):\ 3x+2y+2z=6$
14. $\overline{a}(M)=4x\overline{i}+(x-y-z)\overline{j}+(3y+2z)\overline{k},\qquad (p):\ 2x+y+z=4$
15. $\overline{a}(M)=(2z-x)\overline{i}+(x+2y)\overline{j}+3z\overline{k},\qquad (p):\ x+4y+2z=8$
16. $\overline{a}(M)=4z\overline{i}+(x-y-z)\overline{j}+(3y+z)\overline{k},\qquad (p):\ x-2y+2z=2$
17. $\overline{a}(M)=(x+y)\overline{i}+(y+z)\overline{j}+2(z+x)\overline{k},\qquad (p):\ 3x-2y+2z=6$
18. $\overline{a}(M)=(x+y+z)\overline{i}+2z\overline{j}+(y-7z)\overline{k},\qquad (p):\ 2x+3y+z=6$
19. $\overline{a}(M)=(2x-z)\overline{i}+(y-x)\overline{j}+(x+2z)\overline{k},\qquad (p):\ x-y+z=2$
20. $\overline{a}(M)=(2y-z)\overline{i}+(x+y)\overline{j}+x\overline{k},\qquad (p):\ x+2y+2z=4$
21. $\overline{a}(M)=(2z-x)\overline{i}+(x-y)\overline{j}+(3x+z)\overline{k},\qquad (p):\ x+y+2z=2$
22. $\overline{a}(M)=(x+z)\overline{i}+(x+3y)\overline{j}+y\overline{k},\qquad (p):\ x+y+2z=2$
23. $\overline{a}(M)=(x+z)\overline{i}+z\overline{j}+(2x-y)\overline{k},\qquad (p):\ 2x+2y+z=4$
24. $\overline{a}(M)=(3x+y)\overline{i}+(x+z)\overline{j}+y\overline{k},\qquad (p):\ x+2y+z=2$
25. $\overline{a}(M)=(y+z)\overline{i}+(2x-z)\overline{j}+(y+3z)\overline{k},\qquad (p):\ 2x+y+3z=6$
26. $\overline{a}(M)=(y+z)\overline{i}+(x+6y)\overline{j}+y\overline{k},\qquad (p):\ x+2y+2z=2$
27. $\overline{a}(M)=(2y-z)\overline{i}+(x+2y)\overline{j}+y\overline{k},\qquad (p):\ x+3y+2z=6$
28. $\overline{a}(M)=(y+z)\overline{i}+x\overline{j}+(y-2z)\overline{k},\qquad (p):\ 2x+2y+z=2$
29. $\overline{a}(M)=(x+z)\overline{i}+z\overline{j}+(2x-y)\overline{k},\qquad (p):\ 3x+2y+z=6$
30. $\overline{a}(M)=z\overline{i}+(x+y)\overline{j}+y\overline{k},\qquad (p):\ 2x+y+2z=2$

Задача 14. Обчислити двома способами циркуляцію векторного поля $\overline{a}(M)$ по контуру трикутника, утвореного в результаті перетину площини (p): з координатними площинами, при додатному напрямку обходу контура відносно нормального вектора $\overline{n}$ цієї площини: 1. використовуючи її означення; 2. за допомогою формули Стокса

1. $\overline{a}(M)=z\overline{i}+(x+y)\overline{j}+y\overline{k},\qquad (p):\ 2x+y+2z=2$
2. $\overline{a}(M)=(x+z)\overline{i}+z\overline{j}+(2x-y)\overline{k},\qquad (p):\ 3x+2y+z=6$

3. $\bar{a}(M)=(y+z)\bar{i}+x\bar{j}+(y-2z)\bar{k},$ $(p):\quad 2x+2y+z=2$

4. $\bar{a}(M)=(2y-z)\bar{i}+(x+2y)\bar{j}+y\bar{k},$ $(p):\quad x+3y+2z=6$

5. $\bar{a}(M)=(y+z)\bar{i}+(x+6y)\bar{j}+y\bar{k},$ $(p):\quad x+2y+2z=2$

6. $\bar{a}(M)=(y+z)\bar{i}+(2x-z)\bar{j}+(y+3z)\bar{k},$ $(p):\quad 2x+y+3z=6$

7. $\bar{a}(M)=(3x+y)\bar{i}+(x+z)\bar{j}+y\bar{k},$ $(p):\quad x+2y+z=2$

8. $\bar{a}(M)=(x+z)\bar{i}+z\bar{j}+(2x-y)\bar{k},$ $(p):\quad 2x+2y+z=4$

9. $\bar{a}(M)=(x+z)\bar{i}+(x+3y)\bar{j}+y\bar{k},$ $(p):\quad x+y+2z=2$

10. $\bar{a}(M)=(2y-z)\bar{i}+(x+y)\bar{j}+x\bar{k},$ $(p):\quad x+2y+2z=4$

11. $\bar{a}(M)=(2z-x)\bar{i}+(x-y)\bar{j}+(3x+z)\bar{k},$ $(p):\quad x+y+2z=2$

12. $\bar{a}(M)=(2x-z)\bar{i}+(y-x)\bar{j}+(x+2z)\bar{k},$ $(p):\quad x-y+z=2$

13. $\bar{a}(M)=(x+y+z)\bar{i}+2z\bar{j}+(y-7z)\bar{k},$ $(p):\quad 2x+3y+z=6$

14. $\bar{a}(M)=(x+y)\bar{i}+(y+z)\bar{j}+2(z+x)\bar{k},$ $(p):\quad 3x-2y+2z=6$

15. $\bar{a}(M)=4z\bar{i}+(x-y-z)\bar{j}+(3y+z)\bar{k},$ $(p):\quad x-2y+2z=2$

16. $\bar{a}(M)=(2z-x)\bar{i}+(x+2y)\bar{j}+3z\bar{k},$ $(p):\quad x+4y+2z=8$

17. $\bar{a}(M)=4x\bar{i}+(x-y-z)\bar{j}+(3y+2z)\bar{k},$ $(p):\quad 2x+y+z=4$

18. $\bar{a}(M)=(x+2z)\bar{i}+(y-3z)\bar{j}+z\bar{k},$ $(p):\quad 3x+2y+2z=6$

19. $\bar{a}(M)=x\bar{i}+(y-2z)\bar{j}+(2x-y+2z)\bar{k},$ $(p):\quad x+2y+2z=2$

20. $\bar{a}(M)=(y-z)\bar{i}+(2x+y)\bar{j}+z\bar{k},$ $(p):\quad 2x+y+z=2$

21. $\bar{a}(M)=(x+y-z)\bar{i}-2y\bar{j}+(x+2z)\bar{k},$ $(p):\quad x+2y+z=2$

22. $\bar{a}(M)=(x+y)\bar{i}+3y\bar{j}+(x+z)\bar{k},$ $(p):\quad 2x-y-2z=-2$

23. $\bar{a}(M)=(2y+z)\bar{i}+(x-y)\bar{j}-2z\bar{k},$ $(p):\quad 2x-y+z=2$

24. $\bar{a}(M)=(3x-y)\bar{i}+(2y+z)\bar{j}+(2z-x)\bar{k},$ $(p):\quad 2x-3y+z=6$

25. $\bar{a}(M)=(x+z)\bar{i}+2y\bar{j}+(x+y-z)\bar{k},$ $(p):\quad x+2y+z=2$

26. $\bar{a}(M)=(y+2z)\bar{i}+(x+2z)\bar{j}+(x-2y)\bar{k},$ $(p):\quad 2x+y+2z=2$

27. $\bar{a}(M)=(x+z)\bar{i}+(z-x)\bar{j}+(x+2y+z)\bar{k},$ $(p):\quad x+2y+z=2$

28. $\bar{a}(M)=x\bar{i}+(x+z)\bar{j}+(y+z)\bar{k},$ $(p):\quad 3x+3y+z=3$

29. $\bar{a}(M)=(3x-y)\bar{i}+(y-x+z)\bar{j}+4z\bar{k},$ $(p):\quad 2x-y-2z=2$

30. $\bar{a}(M)=3x\bar{i}+(y+z)\bar{j}+(x-z)\bar{k},$ $(p):\quad x+3y+z=3$

Задача 15. З’ясувати, чи є векторне поле соленоїдним

15.1. $\bar{a}(M)=(\alpha-\beta)x\bar{i}+(\gamma-\alpha)y\bar{j}+(\beta-\gamma)z\bar{k}$

15.2. $\bar{a}(M)=x^2y\bar{i}-2xy^2\bar{j}+2xyz\bar{k}$

15.3. $\bar{a}(M)=(yz-2x)\bar{i}+(xz+2y)\bar{j}+xy\bar{k}$

15.4. $\bar{a}(M)=(x^2-z^2)\bar{i}-3xy\bar{j}+(y^2+z^2)\bar{k}$

15.5. $\bar{a}(M)=2xyz\bar{i}-y(yz+1)\bar{j}+z\bar{k}$

15.6. $\bar{a}(M)=(2x-3y)\bar{i}+2xy\bar{j}-z^2\bar{k}$

15.7. $\bar{a}(M)=(x^2-y^2)\bar{i}+(y^2-z^2)\bar{j}+(z^2-x^2)\bar{k}$

15.8. $\bar{a}(M)=yz\bar{i}+(x-y)\bar{j}+z^2\bar{k}$

15.9. $\bar{a}(M)=(y+z)\bar{i}+(x+z)\bar{j}+(x+y)\bar{k}$

15.10. $\bar{a}(M)=3x^2y\bar{i}-2xy^2\bar{j}-2xyz\bar{k}$

15.11. $\bar{a}(M)=(x+y)\bar{i}-2(y+z)\bar{j}+(z-x)\bar{k}$

15.12. $\bar{a}(M)=(yz-2x)\bar{i}+(xz+zy)\bar{j}+xyz\bar{k}$

15.13. $\bar{a}(M)=yz\bar{i}+xz\bar{j}+xy\bar{k}$

15.14. $\bar{a}(M)=6xy\bar{i}+(3x^2-2y)\bar{j}+z\bar{k}$

15.15. $\bar{a}(M)=(2x-yz)\bar{i}+(2x-xy)\bar{j}+yz\bar{k}$

15.16. $\bar{a}(M)=(y-z)\bar{i}+3xyz\bar{j}+(z-x)\bar{k}$

15.17. $\bar{a}(M)=(y-z)\bar{i}+(x+z)\bar{j}+(x^2-y^2)\bar{k}$

15.18. $\bar{a}(M)=(x+y)\bar{i}-2xz\bar{j}-3(y+z)\bar{k}$

15.19. $\bar{a}(M)=z^2\bar{i}+(xz+y)\bar{j}+x^2y\bar{k}$

15.20. $\bar{a}(M)=xy(3x-4y)\bar{i}+x^2(x-4y)\bar{j}+3z^2\bar{k}$

15.21. $\bar{a}(M)=6x^2\bar{i}+3\cos(3x+2z)\bar{j}+\cos(3y+2z)\bar{k}$

15.22. $\bar{a}(M)=3(x-z)\bar{i}+(x^2+y^2)\bar{j}+3z\bar{k}$

15.23. $\bar{a}(M)=(2x-yz)\bar{i}+(xz-2y)\bar{j}+2xyz\bar{k}$

15.24. $\bar{a}(M)=3x^2\bar{i}+4(x-y)\bar{j}+(x-z)\bar{k}$

15.25. $\bar{a}(M)=(x+y)\bar{i}+(z-y)\bar{j}+2(x+z)\bar{k}$

15.26. $\bar{a}(M)=x^2z\bar{i}+y^2\bar{j}-xz^2\bar{k}$

15.27. $\bar{a}(M)=(x+y)\bar{i}+(y+z)\bar{j}+(x+z)\bar{k}$

15.28. $\bar{a}(M)=xy^{-1}\bar{i}+yz^{-1}\bar{j}+zx^{-1}\bar{k}$

15.29. $\bar{a}(M)=yz\bar{i}+xz\bar{j}+xy\bar{k}$

15.30. $\bar{a}(M)=(y-z)\bar{i}+(z-x)\bar{j}+(x-y)\bar{k}$