

## ТЕМА КРАТНІ ІНТЕГРАЛИ

### Задача 1.Змінити порядок інтегрування

1.  $\int_{-2}^{-1} dy \int_{-\sqrt{2+y}}^0 f(x, y) dx + \int_{-1}^0 dy \int_{-\sqrt{-y}}^0 f(x, y) dx$
2.  $\int_0^1 dy \int_{-\sqrt{y}}^0 f(x, y) dx + \int_1^{\sqrt{2}} dy \int_{-\sqrt{2-y^2}}^0 f(x, y) dx$
3.  $\int_0^1 dy \int_0^y f(x, y) dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f(x, y) dx$
4.  $\int_0^1 dy \int_0^{\sqrt{y}} f(x, y) dx + \int_1^2 dy \int_0^{\sqrt{2-y}} f(x, y) dx$
5.  $\int_{-\sqrt{2}}^{-1} dx \int_{-\sqrt{2-x^2}}^0 f(x, y) dy + \int_{-1}^0 dx \int_x^0 f(x, y) dy$
6.  $\int_0^{\sqrt{2}/2} dy \int_0^{\arcsin y} f(x, y) dx + \int_{\sqrt{2}/2}^1 dy \int_0^{\arccos y} f(x, y) dx$
7.  $\int_{-2}^{-1} dy \int_0^{\sqrt{2+y}} f(x, y) dx + \int_{-1}^0 dy \int_0^{\sqrt{-y}} f(x, y) dx$
8.  $\int_0^1 dy \int_{-\sqrt{y}}^0 f(x, y) dx + \int_1^e dy \int_{-1}^{-\ln y} f(x, y) dx$
9.  $\int_{-\sqrt{2}}^{-1} dx \int_0^{\sqrt{2-x^2}} f(x, y) dy + \int_{-1}^0 dx \int_0^{x^2} f(x, y) dy$
10.  $\int_{-2}^{-\sqrt{3}} dx \int_{-\sqrt{4-x^2}}^0 f(x, y) dy + \int_{-\sqrt{3}}^0 dx \int_{\sqrt{4-x^2}-2}^0 f(x, y) dy$
11.  $\int_0^1 dx \int_{1-x^2}^1 f(x, y) dy + \int_1^e dy \int_{\ln x}^1 f(x, y) dy$
12.  $\int_0^1 dy \int_0^{\sqrt{y}} f(x, y) dx + \int_1^2 dy \int_0^{2-y} f(x, y) dx$
13.  $\int_0^{\pi/4} dy \int_0^{\sin y} f(x, y) dx + \int_{\pi/4}^{\pi/2} dy \int_0^{\cos y} f(x, y) dx$
14.  $\int_{-2}^{-1} dx \int_{-(2+x)}^0 f(x, y) dy + \int_{-1}^0 dx \int_{-\sqrt{-x}}^0 f(x, y) dy$
15.  $\int_0^1 dy \int_0^{\sqrt{y}} f(x, y) dx + \int_1^e dy \int_{\ln y}^1 f(x, y) dx$
16.  $\int_0^1 dy \int_{-\sqrt{y}}^0 f(x, y) dx + \int_1^2 dy \int_{-\sqrt{2-y}}^0 f(x, y) dx$
17.  $\int_0^1 dy \int_{-y}^0 f(x, y) dx + \int_1^{\sqrt{2}} dy \int_{-\sqrt{2-y^2}}^0 f(x, y) dx$
18.  $\int_0^1 dy \int_0^{y^2} f(x, y) dx + \int_1^2 dy \int_0^{2-y} f(x, y) dx$
19.  $\int_0^{\sqrt{3}} dx \int_{\sqrt{4-x^2}-2}^0 f(x, y) dy + \int_{\sqrt{3}}^2 dx \int_{-\sqrt{4-x^2}}^0 f(x, y) dy$
20.  $\int_{-2}^{-1} dy \int_{-(2+y)}^0 f(x, y) dx + \int_{-1}^0 dy \int_{\sqrt{y}}^0 f(x, y) dx$
21.  $\int_0^1 dy \int_0^y f(x, y) dx + \int_1^e dy \int_{\ln y}^1 f(x, y) dx$
22.  $\int_0^1 dx \int_0^{x^2} f(x, y) dy + \int_1^{\sqrt{2}} dx \int_0^{\sqrt{2-x^2}} f(x, y) dy$
23.  $\int_0^{\pi/4} dx \int_0^{\sin x} f(x, y) dy + \int_{\pi/4}^{\pi/2} dx \int_0^{\cos x} f(x, y) dy$
24.  $\int_{-\sqrt{2}}^{-1} dy \int_{-\sqrt{2-y^2}}^0 f(x, y) dx + \int_{-1}^0 dy \int_y^0 f(x, y) dx$
25.  $\int_0^1 dx \int_0^{x^2} f(x, y) dy + \int_1^2 dx \int_0^{2-x} f(x, y) dy$
26.  $\int_0^{\sqrt{3}} dx \int_0^{2-\sqrt{4-x^2}} f(x, y) dy + \int_{\sqrt{3}}^2 dx \int_0^{\sqrt{4-x^2}} f(x, y) dy$
27.  $\int_0^1 dx \int_{-\sqrt{x}}^0 f(x, y) dy + \int_1^2 dx \int_{-\sqrt{2-x}}^0 f(x, y) dy$
28.  $\int_0^1 dx \int_0^x f(x, y) dy + \int_1^{\sqrt{2}} dx \int_0^{\sqrt{2-x^2}} f(x, y) dy$
29.  $\int_0^1 dx \int_0^{\sqrt{x}} f(x, y) dy + \int_1^2 dx \int_0^{\sqrt{2-x}} f(x, y) dy$
30.  $\int_{-2}^{-1} dx \int_{-(2+x)}^0 f(x, y) dy + \int_{-1}^0 dx \int_{-\sqrt{-x}}^0 f(x, y) dy$
31.  $\int_0^1 dy \int_0^{\sqrt{y}} f(x, y) dx + \int_1^2 dy \int_0^{\sqrt{2-y}} f(x, y) dx$
32.  $\int_0^1 dy \int_0^{\sqrt{y}} f(x, y) dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f(x, y) dx$
33.  $\int_{-2}^{-1} dy \int_0^{\sqrt{2+y}} f(x, y) dx + \int_{-1}^0 dy \int_0^{\sqrt{-y}} f(x, y) dx$

**Задача 2** Розставити границі інтегрування двома способами, якщо область D подана зазначеними лініями

1.  $D: y = \sqrt{4-x^2}; y = \sqrt{3x}; x \geq 0$
2.  $D: x^2 = 2y; 5x - 2y - 6 = 0$
3.  $D: x = \sqrt{8-y^2}; y \geq 0; y = x$
4.  $D: x \geq 0; y \geq 0; y \leq 1; y = \ln x$
5.  $D: x^2 = 2 - y; x + y = 0$
6.  $D: y = \sqrt{2-x^2}; y = x^2$
7.  $D: x \geq 0; y \geq 0; y = 1; x = \sqrt{4-y^2}$
8.  $D: y = x^2 - 2; y = x$
9.  $D: x \geq 0; y \geq 1; y \leq 3; y = x$
10.  $D: y^2 = 2x; x^2 = 2y; x \leq 1$
11.  $D: x \geq 0; y \geq x; y = \sqrt{8-x^2}$
12.  $D: y^2 = 2 - x; y = x$
13.  $D: x = \sqrt{2-y^2}; x = y^2; y \geq 0$
14.  $D: y \geq 0; x + 2y - 12 = 0; y = x$
15.  $D: x \leq 0; y \geq 1; y \leq 3; y = -x$
16.  $D: y = 0; y \geq x; y = -\sqrt{2-x^2}$
17.  $D: y \geq 0; x = \sqrt{y}; y = \sqrt{6-x^2}$
18.  $D: y = -x; y^2 = x + 3$
19.  $D: y = \sqrt{4-x^2}; x \geq 0; x = 1; y = 0$
20.  $D: x = -1; x = -2; y \geq 0; y = x^2$
21.  $D: y \leq 0; x^2 = -y; x = \sqrt{2-y^2}$
22.  $D: y \geq 0; y \leq 1; y = x; x^2 + y^2 = 4$
23.  $D: y = 6 - x^2; y = -x$
24.  $D: x = 0; x = -2; y \geq 0; y = x^2 + 4$
25.  $D: x = 0; y = 0; y = 1; (x-3)^2 + y^2 = 1$
26.  $D: x = \sqrt{9-y^2}; y = x; y \geq 0$
27.  $D: x \leq 0; y = 1; y = 4; y = -x$
28.  $D: x + 2y - 6 = 0; y = x; y \geq 0$
29.  $D: y = -x; 3x + y = 3; y = 3$
30.  $D: x \geq 0; y = 1; y = -1; y = \log_{1/2} x$
31.  $D: y \geq 0; x = \sqrt{y}; y = \sqrt{6-x^2}$
32.  $D: y = \sqrt{2-x^2}; y = x^2$
33.  $D: x = \sqrt{2-y^2}; x = y^2; y \geq 0$

**Задача 3** Обчислити подвійний інтеграл по області D, обмеженої вказаними лініями

|                              |                                      |                                |   |                                |   |
|------------------------------|--------------------------------------|--------------------------------|---|--------------------------------|---|
| 1. $\iint (x^2 + y) dx dy,$  | $D: x^2 = y; y^2 = x$                | 12. $\iint x^2 y dx dy,$       | $D: y = 2x^2; y = 0; x = 1$             | 23. $\iint (x^3 + y) dx dy,$   | $D: x + y = 1; x + y = 2; x \leq 1; x \geq 0$ |
| 2. $\iint xy^2 dx dy,$       | $D: y = x^2; y = 2x$                 | 13. $\iint (x^2 + y^2) dx dy,$ | $D: x = y^2; x = 1$                     | 24. $\iint xy^3 dx dy,$        | $D: y = x^3; y \geq 0; y = 4x$                |
| 3. $\iint (x + y) dx dy,$    | $D: y^2 = x; y = x$                  | 14. $\iint xy dx dy,$          | $D: y = x^3; y = 0; x \leq 2$           | 25. $\iint (x^3 + 3y) dx dy,$  | $D: x + y = 1; y = x^2 - 1; x \geq 0$         |
| 4. $\iint x^2 y dx dy,$      | $D: y = 2 - x; y = x; x \geq 0$      | 15. $\iint (x + y) dx dy,$     | $D: y = x^3; y = 8; y = 0; x = 3$       | 26. $\iint xy dx dy,$          | $D: y = \sqrt{x}; y = 0; x + y = 2; x = 0$    |
| 5. $\iint (x^3 - 2y) dx dy,$ | $D: y = x^2 - 1; x \geq 0; y \leq 0$ | 16. $\iint x(2x + y) dx dy,$   | $D: y = 1 - x^2; y \geq 0$              | 27. $\iint y^2 x^{-2} dx dy$   | $D: y = x; xy = 1; y = 2$                     |
| 6. $\iint (y - x) dx dy,$    | $D: y = x; y = x^2$                  | 17. $\iint y(1 - x) dx dy,$    | $D: y^3 = x; y = x$                     | 28. $\iint y(1 + x^2) dx dy,$  | $D: y = x^3; y = 3x$                          |
| 7. $\iint (1 + y) dx dy,$    | $D: y^2 = x; 5y = x$                 | 18. $\iint xy^3 dx dy,$        | $D: y^2 = 1 - x; x \geq 0$              | 29. $\iint y^2(1 + 2x) dx dy,$ | $D: x = 2 - y^2; x = 0$                       |
| 8. $\iint (x + y) dx dy,$    | $D: y = x^2 - 1; y = -x^2 + 1$       | 19. $\iint x(y + 5) dx dy,$    | $D: y = x + 5; x + y + 5 = 0; x \leq 0$ | 30. $\iint e^y dx dy,$         | $D: y = \ln x; y = 0; x = 2$                  |
| 9. $\iint x(y - 1) dx dy,$   | $D: y = 5x; y = x; x = 3$            | 20. $\iint (x - y) dx dy,$     | $D: y = x^2 - 1; y = 3$                 | 31. $\iint x(2x + y) dx dy,$   | $D: y = x^2 - 1; y = 3$                       |
| 10. $\iint (x - 2)y dx dy,$  | $D: y = x; y = 0,5x; x = 2$          | 21. $\iint (x + 1)y^2 dx dy,$  | $D: y = 3x^2; y = 3$                    | 32. $\iint (x - 2)y dx dy,$    | $D: x = y^2; x = 1$                           |
| 11. $\iint (x - y^2) dx dy,$ | $D: y = x^2; y = 1$                  | 22. $\iint xy^2 dx dy,$        | $D: y = x; y = 0; x = 1$                | 33. $\iint x(y - 1) dx dy,$    | $D: y = x^2; y = 2x$                          |

**Задача 4** Обчислити подвійний інтеграл у полярних координатах

|  |  |   |   |  |
|--|--|---|---|--|
| 1. $\int_0^3 dx \int_0^{\sqrt{9-x^2}} \ln(1+x^2+y^2) dy$   | 2. $\int_{-\sqrt{2}}^{\sqrt{2}} dx \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} e^{-(x^2+y^2)} dy$                            | 3. $\int_{-1}^1 dx \int_0^{\sqrt{1-x^2}} \sqrt{1+x^2+y^2} dy$   | 4. $\int_0^1 dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{\ln(1+\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}} dy$      | 5. $\int_{-3}^3 dx \int_{-\sqrt{9-x^2}}^0 \frac{xy}{x^2+y^2} dy$   |
| 6. $\int_0^R dx \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \frac{dy}{\sqrt{x^2+y^2} \cos^2 \sqrt{x^2+y^2}}$             | 7. $\int_0^2 dx \int_0^{\sqrt{4-x^2}} \cos \sqrt{x^2+y^2} dy$  | 8. $\int_{-2}^2 dx \int_0^{\sqrt{4-x^2}} (x^2+y^2) e^{x^2+y^2} dy$  | 9. $\int_0^2 dx \int_0^{\sqrt{4-x^2}} \frac{dy}{1+x^2+y^2}$   | 10. $\int_{-R}^0 dx \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \frac{dy}{\sqrt{x^2+y^2} \operatorname{ctg} \sqrt{x^2+y^2}}$ |
| 11. $\int_0^R dx \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \sin(x^2+y^2) dy$   | 12. $\int_0^R dx \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \frac{\operatorname{tg} \sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} dy$ | 13. $\int_{-\sqrt{2}}^{\sqrt{2}} dx \int_{\sqrt{2-x^2}}^{\sqrt{2-x^2}} (1+x^2+y^2) dy$                    | 14. $\int_0^R dx \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \frac{dy}{\sqrt{x^2+y^2} \sin^2 \sqrt{x^2+y^2}}$ | 15. $\int_0^2 dx \int_0^{\sqrt{4-x^2}} \frac{xy}{\sqrt{x^2+y^2}} dy$   |
| 16. $\int_{-R}^0 dx \int_0^{\sqrt{R^2-x^2}} \sin(x^2+y^2) dy$  | 17. $\int_{-R}^R dx \int_{-\sqrt{R^2-x^2}}^0 \frac{\sin \sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} dy$                          | 18. $\int_{-R}^0 dx \int_{-\sqrt{R^2-x^2}}^0 \cos(x^2+y^2) dy$  | 19. $\int_0^1 dx \int_0^{\sqrt{1-x^2}} \frac{dy}{1+\sqrt{x^2+y^2}}$                                       | 20. $\int_{-R}^R dx \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \sin \sqrt{x^2+y^2} dy$                                      |
| 21. $\int_{-2}^2 dx \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{1+x^2+y^2} dy$   | 22. $\int_{-\sqrt{3}}^{\sqrt{3}} dx \int_0^{\sqrt{3-x^2}} \sqrt[3]{1+x^2+y^2} dy$                                    | 23. $\int_0^1 dx \int_0^{\sqrt{1-x^2}} \ln(1+x^2+y^2) dy$   | 24. $\int_0^R dx \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \cos(x^2+y^2) dy$                                | 25. $\int_{-R}^R dx \int_0^{\sqrt{R^2-x^2}} \operatorname{tg}(x^2+y^2) dy$   |
| 26. $\int_0^R dx \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \frac{\operatorname{tg} \sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} dy$ | 27. $\int_{-R}^0 dx \int_0^{\sqrt{R^2-x^2}} \cos \sqrt{x^2+y^2} dy$  | 28. $\int_{-\sqrt{3}}^0 dx \int_0^{\sqrt{3-x^2}} \frac{dy}{\sqrt{1+x^2+y^2}}$                             | 29. $\int_{-\sqrt{2}}^{\sqrt{2}} dx \int_{-\sqrt{2-x^2}}^0 \frac{xy}{x^2+y^2} dy$                         | 30. $\int_0^1 dx \int_0^{\sqrt{1-x^2}} \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}} dy$  |
| 31. $\int_{-3}^3 dx \int_{-\sqrt{9-x^2}}^0 \frac{xy}{x^2+y^2} dy$  | 32. $\int_{-R}^R dx \int_{-\sqrt{R^2-x^2}}^0 \frac{\sin \sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} dy$                          | 33. $\int_0^R dx \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \frac{dy}{\sqrt{x^2+y^2} \sin^2 \sqrt{x^2+y^2}}$ | 34. $\int_0^R dx \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \sin(x^2+y^2) dy$                                | 35. $\int_{-3}^3 dx \int_{-\sqrt{9-x^2}}^0 \frac{xy}{x^2+y^2} dy$  |

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| Задача 5 | Розставити границі інтегрування в потрібному інтегралі $\iiint_V f(x,y,z) dx dy dz$ , якщо область $V$ обмежена зазначеними поверхнями. Побудувати область інтегрування |
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1.  $x=1, y=3x, y \geq 0, z \geq 0, z=2(x^2+y^2)$
2.  $x=1, y=4x, z \geq 0, z=\sqrt{3y}$
3.  $x=3, y=x, y \geq 0, z \geq 0, z=3x^2+y^2$
4.  $y=2x, y=2, z \geq 0, z=2\sqrt{x}$

5.  $x=0, y=x, y=5, z \geq 0, z=2x^2+y^2$
6.  $x \geq 0, y=2x, y=1, z \geq 0, x+y+z=3$
7.  $x \geq 0, y=3x, y=3, z \geq 0, x=3\sqrt{z}$
8.  $x=5, y=0,2x, y \geq 0, z \geq 0, z=x^2+5y^2$

9.  $x=2, y=4x, y=3\sqrt{x}, z \geq 0, z=4$
10.  $x=2, y=4x, z \geq 0, y=2\sqrt{z}$
11.  $x=3, y=x/3, y \geq 0, z \geq 0, 2z=x^2+y^2$
12.  $x=4, y=x/4, z \geq 0, z=4y^2$

$$13. x \geq 0, y = 3x, y = 3, z \geq 0, z = 2(x^2 + y^2)$$

$$14. x \geq 0, y = 4x, y = 8, z \geq 0, z = 3x^2 + y^2$$

$$15. x \geq 0, y = 5x, y = 10, z \geq 0, z = x^2 + y^2$$

$$16. y = x, y = -x, y = 2, z \geq 0, z = 3(x^2 + y^2)$$

$$17. x = 1, y = 2x, y = 3x, z \geq 0, z = 2x^2 + y^2$$

$$18. y = x, y = -2x, y = 1, z \geq 0, z = x^2 + 4y^2$$

$$19. x \geq 0, y \geq 0, z \geq 0, x + y = 1, z = 3x^2 + 2y^2$$

$$20. x \geq 0, y \geq 0, z \geq 0, 3x + 2y = 6, z = x^2 + y^2$$

$$21. x \geq 0, y \geq 0, z \geq 0, x + y = 2, z = 4 - x^2 - y^2$$

$$22. x \geq 0, y \geq 0, z \geq 0, x + y = 3, z = 9 - x^2 - y^2$$

$$23. x \geq 0, y \geq 0, z \geq 0, 3x + 4y = 12, z = 6 - x^2 - y^2$$

$$24. x \geq 0, y = x, z \geq 0, y = 3, z = 18 - x^2 - y^2$$

$$25. x = 2, y \geq 0, z \geq 0, y = 3x, z = 4(x^2 + y^2)$$

$$26. x \geq 0, y = 2x, y = 4, z \geq 0, z = 10 - x^2 - y^2$$

$$27. x = 3, y \geq 0, z \geq 0, y = 2x, z = 4\sqrt{y}$$

$$28. x \geq 0, y \geq 0, z \geq 0, 2x + 3y = 6, z = 3 + x^2 + y^2$$

$$29. x \geq 0, y \geq 0, z \geq 0, x + y = 4, z = 16 - x^2 - y^2$$

$$30. y = x, y = -x, y = 2, z \geq 0, z = 3(x^2 + y^2)$$

$$31. x = 5, y = 0, 2x, y \geq 0, z \geq 0, z = x^2 + 5y^2$$

$$32. x \geq 0, y = 3x, y = 3, z \geq 0, z = 2(x^2 + y^2)$$

$$33. x \geq 0, y = 2x, y = 1, z \geq 0, x + y + z = 3$$

$$34. x \geq 0, y \geq 0, z \geq 0, 5x + y = 5, z = x^2 + y^2$$

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| Задача 6 | Обчислити потрійний інтеграл від функції $F(x, y, z)$ по області $V$ |
|----------|--|

|     | $F(x, y, z)$      | $V$                 |                     |                    |
|-----|-------------------|---------------------|---------------------|--------------------|
| 1.  | $2x^2 + 3y + z,$  | $2 \leq x \leq 3$   | $-1 \leq y \leq 2,$ | $0 \leq z \leq 4$  |
| 2.  | $x^2 yz$          | $-1 \leq x \leq 2,$ | $0 \leq y \leq 3,$  | $2 \leq z \leq 3$  |
| 3.  | $x + y + 4z^2$    | $-1 \leq x \leq 1,$ | $0 \leq y \leq 2,$  | $-1 \leq z \leq 1$ |
| 4.  | $x^2 + y^2 + z^2$ | $0 \leq x \leq 3,$  | $-1 \leq y \leq 2,$ | $0 \leq z \leq 2$  |
| 5.  | $x^2 y^2 z$       | $-1 \leq x \leq 3,$ | $0 \leq y \leq 2,$  | $-2 \leq z \leq 5$ |
| 6.  | $x + y + z$       | $0 \leq x \leq 1,$  | $-1 \leq y \leq 0,$ | $1 \leq z \leq 2$  |
| 7.  | $2x - y^2 - z$    | $1 \leq x \leq 5,$  | $0 \leq y \leq 2,$  | $-1 \leq z \leq 0$ |
| 8.  | $2xy^2 z$         | $0 \leq x \leq 3,$  | $-2 \leq y \leq 0,$ | $1 \leq z \leq 2$  |
| 9.  | $5xyz^2$          | $-1 \leq x \leq 0,$ | $2 \leq y \leq 3,$  | $1 \leq z \leq 2$  |
| 10. | $x^2 + 2y^2 - z$  | $0 \leq x \leq 1,$  | $0 \leq y \leq 3,$  | $-1 \leq z \leq 2$ |
| 11. | $x + 2yz$         | $-2 \leq x \leq 0,$ | $0 \leq y \leq 1,$  | $0 \leq z \leq 2$  |
| 12. | $x + yz^2$        | $0 \leq x \leq 1,$  | $0 \leq y \leq 2,$  | $-1 \leq z \leq 3$ |
| 13. | $xy + 3z$         | $-1 \leq x \leq 1,$ | $0 \leq y \leq 1,$  | $1 \leq z \leq 2$  |
| 14. | $xy - z^2$        | $0 \leq x \leq 2,$  | $0 \leq y \leq 1,$  | $-1 \leq z \leq 3$ |
| 15. | $x^3 + yz$        | $-1 \leq x \leq 2,$ | $0 \leq y \leq 1,$  | $0 \leq z \leq 1$  |
| 16. | $x^3 + y^2 - z$   | $0 \leq x \leq 2,$  | $-1 \leq y \leq 0,$ | $0 \leq z \leq 1$  |

|     |                  |                     |                     |                    |
|-----|------------------|---------------------|---------------------|--------------------|
| 17. | $2x^2 + y - z^3$ | $0 \leq x \leq 1,$  | $-2 \leq y \leq 1,$ | $0 \leq z \leq 1$  |
| 18. | $x^2 yz^2$       | $0 \leq x \leq 2,$  | $1 \leq y \leq 2,$  | $-1 \leq z \leq 0$ |
| 19. | $x + y - z$      | $0 \leq x \leq 4,$  | $1 \leq y \leq 3,$  | $-1 \leq z \leq 5$ |
| 20. | $x + 2y + 3z^2$  | $-1 \leq x \leq 2,$ | $0 \leq y \leq 1,$  | $1 \leq z \leq 2$  |
| 21. | $3x^2 + 2y + z$  | $0 \leq x \leq 1,$  | $0 \leq y \leq 1,$  | $-1 \leq z \leq 3$ |
| 22. | $xy - z^3$       | $0 \leq x \leq 1,$  | $-1 \leq y \leq 2,$ | $0 \leq z \leq 3$  |
| 23. | $x^3 yz$         | $-1 \leq x \leq 2,$ | $1 \leq y \leq 3,$  | $0 \leq z \leq 1$  |
| 24. | $xy^2 z$         | $-2 \leq x \leq 1,$ | $0 \leq y \leq 2,$  | $0 \leq z \leq 3$  |
| 25. | $xyz^2$          | $0 \leq x \leq 2,$  | $-1 \leq y \leq 0,$ | $0 \leq z \leq 4$  |
| 26. | $x + yz$         | $0 \leq x \leq 1,$  | $-1 \leq y \leq 4,$ | $0 \leq z \leq 2$  |
| 27. | $x + y^2 - z^2$  | $-2 \leq x \leq 0,$ | $1 \leq y \leq 2,$  | $0 \leq z \leq 5$  |
| 28. | $x + y + z^2$    | $-1 \leq x \leq 0,$ | $0 \leq y \leq 1,$  | $2 \leq z \leq 3$  |
| 29. | $x + y^2 - 2z$   | $1 \leq x \leq 2,$  | $-2 \leq y \leq 3,$ | $0 \leq z \leq 1$  |
| 30. | $x - y - z$      | $0 \leq x \leq 3,$  | $0 \leq y \leq 1,$  | $-2 \leq z \leq 1$ |
| 31. | $xyz^2$          | $-1 \leq x \leq 2,$ | $-1 \leq y \leq 0,$ | $0 \leq z \leq 1$  |
| 32. | $3x^2 + 2y + z$  | $0 \leq x \leq 2,$  | $2 \leq y \leq 3,$  | $-1 \leq z \leq 0$ |
| 33. | $x^3 yz$         | $-1 \leq x \leq 2,$ | $0 \leq y \leq 1,$  | $0 \leq z \leq 5$  |

34.  $x - y - z$                        $0 \leq x \leq 1, \quad -1 \leq y \leq 0, \quad 0 \leq z \leq 5$

|          |   |
|----------|---|
| Задача 7 | Обчислити потрійний інтеграл від функції $F(x, y, z)$ по області $V$ за допомогою циліндричної або сферичної систем координат |
|----------|---|

- |     | $F(x, y, z)$                             | $V$  |     |                                    |   |
|-----|--|--|-----|------------------------------------|---|
| 1.  | $x^2 + y^2 + z^2$                        | $x^2 + y^2 + z^2 = 4; x \geq 0; y \geq 0; z \geq 0$                    | 16. | $\sqrt{x^2 + y^2}$                 | $x^2 + y^2 = 2x; x + z = 2; z \geq 0$   |
| 2.  | $y\sqrt{x^2 + y^2}$                      | $0 \leq z \leq 2; -x \leq y \leq x; z^2 = 4(x^2 + y^2)$                | 17. | $xy$                               | $2 \leq x^2 + y^2 + z^2 \leq 8; z^2 = x^2 + y^2; x \geq 0; y \geq 0; z \geq 0$  |
| 3.  | $z^2$                                    | $1 \leq x^2 + y^2 \leq 36; y \geq x; x \geq 0; z \geq 0$               | 18. | $\frac{y}{\sqrt{x^2 + y^2}}$       | $x^2 + y^2 = 2y; x^2 + y^2 = 4y; x \geq 0; z \geq 0; z = 6$                     |
| 4.  | $y$                                      | $x^2 + y^2 + z^2 = 32; y^2 = x^2 + z^2; y \geq 0$                      | 19. | $\sqrt{x^2 + y^2 + z^2}$           | $x^2 + y^2 + z^2 = 36; y \geq 0; z \geq 0; y \leq -x$                           |
| 5.  | $x$                                      | $x^2 + y^2 + z^2 = 8; x^2 = y^2 + z^2; x \geq 0$                       | 20. | $\frac{x}{\sqrt{x^2 + y^2}}$       | $x^2 + y^2 = 2x; x^2 + y^2 = 4x; y \geq 0; z \geq 0; z = 4; y \leq x$           |
| 6.  | $y$                                      | $4 \leq x^2 + y^2 + z^2 \leq 16; y \leq \sqrt{3}x; y \geq 0; z \geq 0$ | 21. | $\frac{z}{\sqrt{x^2 + y^2 + z^2}}$ | $1 \leq x^2 + y^2 + z^2 \leq 9; y \geq 0; y \leq \frac{1}{\sqrt{3}}x; z \geq 0$ |
| 7.  | $y$                                      | $z = \sqrt{8 - x^2 - y^2}; z = \sqrt{x^2 + y^2}; y \geq 0$             | 22. | $\sqrt{x^2 + y^2}$                 | $x^2 - 2x + y^2 = 0; y \geq 0; z \geq 0; x + z = 2$                             |
| 8.  | $\frac{y^2}{x^2 + y^2 + z^2}$            | $x \geq 0; y \geq \sqrt{3}x; z \geq 0; 4 \leq x^2 + y^2 + z^2 \leq 36$ | 23. | $x^2$                              | $1 \leq x^2 + y^2 + z^2 \leq 16; y \geq 0; z \geq 0; y \leq x$                  |
| 9.  | $\frac{y^2 z}{\sqrt{(x^2 + y^2)^3}}$     | $y \geq 0; y \leq \sqrt{3}x; z = 3(x^2 + y^2); z = 3$                  | 24. | $\frac{1}{\sqrt{x^2 + y^2}}$       | $x^2 + y^2 = 4y; y + z = 4; z \geq 0$   |
| 10. | $\frac{x^2}{\sqrt{(x^2 + y^2 + z^2)^3}}$ | $x^2 + y^2 + z^2 = 16; z \geq 0$                                       | 25. | $\frac{z}{\sqrt{x^2 + y^2 + z^2}}$ | $4 \leq x^2 + y^2 + z^2 \leq 16; y \leq \sqrt{3} \cdot x; y \geq 0; z \geq 0$   |
| 11. | $\frac{xz}{\sqrt{x^2 + y^2}}$            | $z = 2(x^2 + y^2); y \geq 0; y \leq \frac{1}{\sqrt{3}}x; z = 18$       | 26. | $z\sqrt{x^2 + y^2}$                | $x^2 + y^2 = 2x; y \geq 0; z \geq 0; z = 3$                                     |
| 12. | $\frac{xy}{\sqrt{(x^2 + y^2)^3}}$        | $z = x^2 + y^2; y \geq 0; y \leq x; z = 4$                             | 27. | $\frac{x}{\sqrt{x^2 + y^2 + z^2}}$ | $1 \leq x^2 + y^2 + z^2 \leq 4; x \geq 0; y \geq 0; z \geq 0; y \leq x$         |
| 13. | $\frac{z}{\sqrt{x^2 + y^2}}$             | $x^2 + y^2 = 4y; y + z = 4; z \geq 0$                                  | 28. | $x$                                | $x^2 = 2(y^2 + z^2); x = 4; x \geq 0$   |
| 14. | $\frac{y}{\sqrt{x^2 + y^2}}$             | $x^2 + y^2 = 2x; x + z = 2; z \geq 0; y \geq 0$                        | 29. | $\frac{y}{\sqrt{x^2 + y^2 + z^2}}$ | $1 \leq x^2 + y^2 + z^2 \leq 9; y \geq 0; z \geq 0; y \leq x$                   |
| 15. | $\frac{x}{\sqrt{x^2 + y^2}}$             | $x^2 + y^2 = 16y; y + z = 16; z \geq 0; x \geq 0$                      | 30. | $x$                                | $z = \sqrt{18 - x^2 - y^2}; z = \sqrt{x^2 + y^2}; x \geq 0$                     |

31.  $\frac{z}{\sqrt{x^2 + y^2 + z^2}}$   $1 \leq x^2 + y^2 + z^2 \leq 4$ ;  $x \geq 0$ ;  $y \geq 0$ ;  $z \geq 0$ ;  $y \leq x$

32.  $\frac{x}{\sqrt{x^2 + y^2 + z^2}}$   $1 \leq x^2 + y^2 + z^2 \leq 16$ ;  $y \geq 0$ ;  $z \geq 0$ ;  $y \leq x$

33.  $\frac{x}{\sqrt{x^2 + y^2}}$   $x^2 + y^2 = 2x$ ;  $x + z = 2$ ;  $z \geq 0$ ;  $y \geq 0$

34.  $\frac{1}{\sqrt{x^2 + y^2}}$   $x^2 + y^2 = 2y$ ;  $x^2 + y^2 = 4y$ ;  $x \geq 0$ ;  $z \geq 0$ ;  $z = 6$